

Review of Vectors and Matrices

1. Vectors

A **vector** (or n -vector) is an n -tuple of numbers; they are usually real numbers, but we will sometimes allow them to be complex numbers. All the rules and operations below apply just as well to n -tuples of complex numbers. (In the context of vectors, a single real or complex number, i.e., a constant, is called a **scalar**.) As we are dealing with 2×2 linear systems, we are primarily interested in scalars and 2-vectors: ordered pairs of numbers.

The pair can be written horizontally as a **row vector** or vertically as a **column vector**. In these notes, it will almost always be a column. To save space, we will sometimes write the column vector as shown below; the small T stands for **transpose**, and means: change the row to a column.

$$\mathbf{a} = (a, b) \quad \text{row vector} \qquad \mathbf{a} = (a, b)^T \quad \text{column vector}$$

These notes use boldface for vectors here; in handwriting, place an arrow $\vec{a}$ over the letter.

Vector operations. Here are two standard operations on vectors:

- addition: $(a, b) + (c, d) = (a + c, b + d)$.
- multiplication by a scalar: $c(a, b) = (ca, cb)$
- scalar product: $(a, b)(c, d) = ac + bd$

2. Matrices

An $m \times n$ **matrix** A is a rectangular array of numbers (real or complex) having m rows and n columns. The element in the i -th row and j -th column is called the **ij -th entry** and written a_{ij} . The matrix itself is sometimes written (a_{ij}) , i.e., by giving its generic entry, inside the matrix parentheses. We will be interested in matrices where m and n are at most 2.

Note that a 1×2 matrix is a row vector; an 2×1 matrix is a column vector.

Matrix operations.

- addition: if A and B are both $m \times n$ matrices, they are added by adding the corresponding entries; i.e., if $A = (a_{ij})$ and $B = (b_{ij})$, then $A + B = (a_{ij} + b_{ij})$.

- multiplication by a scalar: to get cA , multiply every entry of A by the scalar c ; i.e., if $A = (a_{ij})$, then $cA = (ca_{ij})$.
- matrix multiplication: if A is an $m \times n$ matrix and B is an $n \times k$ matrix, their product AB is an $m \times k$ matrix, defined by using the scalar product operation:

$$ij\text{-th entry of } AB = (i\text{-th row of } A)(j\text{-th column of } B)^T$$

where the scalar product of two 1-vectors is just their normal product.

The definition makes sense since both vectors on the right are vectors of the same length n . In what follows, the most important cases of matrix multiplication will be:

- A and B are square 2×2 matrices. In this case, multiplication is always possible, and the product AB is again an 2×2 matrix.
- A is an 2×2 matrix and $B = \mathbf{b}$, a column 2-vector. In this case, the matrix product $A\mathbf{b}$ is again a column 2-vector.

Laws satisfied by the matrix operations.

For any matrices for which the products and sums below are defined, we have

$$\begin{array}{lll} (AB)C & = & A(BC) & \text{(associative law)} \\ A(B+C) & = & AB + AC, \quad (A+B)C = AB + AC & \text{(distributive laws)} \\ AB & \neq & BA & \text{(commutative law fails in general)} \end{array}$$

The **identity matrix** I is the 2×2 matrix with 1's on the main diagonal (upper left and bottom right), and 0's elsewhere. If A is an arbitrary 2×2 matrix, it is easy to check from the definition of matrix multiplication that

$$AI = A \quad \text{and} \quad IA = A.$$

The exercises later in this session should help you get familiar with all these concepts.

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